A Local Realistic Theory for Mermin's EPR Experiment

Boon Leong Lan

Abstract As another counterexample to prevalent conventional belief, a realistic theory, which is local and reproduces all the probabilistic predictions of quantum theory, is presented for Mermin's version of the Einstein-Podolsky-Rosen (EPR) experiment.

Recent Einstein-Podolsky-Rosen (EPR) experiments [1, 2] have further confirmed the predictions of quantum theory instead of the predictions of local realistic theory à la John Bell [3, 4]. This has led to the widespread conventional belief [5] that no theory that is simultaneously local and realistic can reproduce the quantum mechanical predictions for the experiment. However, contrary to conventional belief, local realistic theories [6–10] that do reproduce the quantum mechanical results for the EPR experiment have been published recently. In this paper, as another counterexample to conventional belief, I present a realistic theory for the EPR experiment based on the realistic interpretation [11, 12] of the EPR state and show that the theory is local and consistent with all the probabilistic predictions of quantum theory. This local realistic theory is similar in spirit to those for the Greenberger-Horne-Zeilinger (GHZ) experiment [13] and Hardy's experiment [14].

In the EPR experiment, a source produces a system of two spin-1/2 particles (labeled left and right respectively) that fly apart in opposite directions, each towards a Stern-Gerlach magnet. Each magnet can be rotated in a plane perpendicular to the line of flight of the particles. For simplicity, I will consider Mermin's [15, 16] version of the EPR experiment where each magnet has only three possible angle settings: a, b, and c. There are thus nine possible pairs of magnet settings (thus nine possible joint measurements on the system of two particles): aa, ab, ac, ba, bb, bc, ca, cb, and cc. For example, magnet settings ab means that the angle of the left magnet is a and the angle of the right magnet is b.

B.L. Lan (✉)
School of Engineering, Monash University, 46150 Sunway, Selangor, Malaysia
e-mail: lan.boon.leong@eng.monash.edu.my

Table 1 A possible set of nine different pre-existing system spin states, one state for each possible pair of magnet settings (one state for each possible joint measurement), of the system of two spin-1/2 particles in an experimental run

Possible pair of magnet settings	Pre-existing system spin state
aa	$|+a\rangle_L|-a\rangle_R$
ab	$|-a\rangle_L|-b\rangle_R$
ac	$|-a\rangle_L|-c\rangle_R$
ba	$|-b\rangle_L|-a\rangle_R$
bb	$|+b\rangle_L|-b\rangle_R$
bc	$|+b\rangle_L|+c\rangle_R$
ca	$|+c\rangle_L|+a\rangle_R$
cb	$|+c\rangle_L|+b\rangle_R$
cc	$|-c\rangle_L|+c\rangle_R$

The system of two spin-1/2 particles is quantum mechanically described by the EPR entangled state which can be expressed in the basis states of any of the possible joint measurements [17]:

$$|\psi\rangle = c_{+\theta+\phi}\,|+\theta\rangle_L\,|+\phi\rangle_R + c_{+\theta-\phi}\,|+\theta\rangle_L\,|-\phi\rangle_R + c_{-\theta+\phi}\,|-\theta\rangle_L\,|+\phi\rangle_R + c_{-\theta-\phi}\,|-\theta\rangle_L\,|-\phi\rangle_R \tag{1}$$

where θ, ϕ is a, a or a, b or a, c or b, a or b, b or b, c or c, a or c, b or c, c. For a given pair of magnet settings, the amplitude square of each coefficient gives the quantum probability of the corresponding joint measurement outcome. For example, if the magnet settings is aa, the probability of measuring $+a$ (spin up along a) for the left (L) particle and $+a$ (spin up along a) for the right (R) particle is $|c_{+a+a}|^2$.

According to the realistic interpretation of the EPR state $|\psi\rangle$ in (1), prior to measurement in an experimental run, the system of two spin-1/2 particles has nine different system spin states (these are spin states of the system as a *whole*), one state for each of the nine possible pairs of magnet settings (one state for each of the nine possible joint measurements). In particular, for each of the nine possible pairs of magnet settings, the corresponding pre-existing system spin state is one of the basis states in the expression for the EPR state $|\psi\rangle$. For example, for the possible aa magnet settings, the corresponding pre-existing system spin state is either $|+a\rangle_L|+a\rangle_R$ or $|+a\rangle_L|-a\rangle_R$ or $|-a\rangle_L|+a\rangle_R$ or $|-a\rangle_L|-a\rangle_R$. The nine different pre-existing system spin states are determined by hidden variables in the source. Table 1 lists a possible set of nine pre-existing system spin states which exists from the moment the system of two particles are produced at the source. In this example, the nine different pre-existing spin states of the system of two particles are $+a$ and $-a$, $-a$ and $-b$, $-a$ and $-c$, $-b$ and $-a$, $+b$ and $-b$, $+b$ and $+c$, $+c$ and $+a$, $+c$ and $+b$, $-c$ and $+c$.

According to the realistic interpretation, in an experimental run, the joint measurement merely reveals the pre-existing system spin state corresponding to the chosen pair of magnet settings. For example, suppose the nine pre-existing system spin states in an experimental run are those in Table 1. If the magnet settings are chosen to be aa, the joint measurement outcome is $+a$ (for the left particle) and $-a$ (for the right particle), which reveals that the corresponding pre-existing system spin state is $|+a\rangle_L|-a\rangle_R$. However, if the magnet settings are chosen to be ab instead in the same experimental run, the joint measurement will yield $-a$ (for the left particle) and $-b$ (for the right particle), revealing that the pre-existing system spin state corresponding to this pair of magnet settings is $|-a\rangle_L|-b\rangle_R$.

In contrast, according to the orthodox interpretation [16] of the experiment, the system of two particles does not have any definite spin state before measurement. Measurement on one

particle compels that particle to acquire a definite spin state and instantaneously 'triggers' the other particle, which is spatially separated from the first, to also acquire a definite spin state. This *instantaneous action-at-a-distance*, i.e., instantaneous triggering of state acquisition over a distance, is, according to Bell [18] and also Mermin [16], *non-locality*. However, in the realistic interpretation presented above, there is no action-at-a-distance because the joint measurement merely reveals the pre-existing system spin state corresponding to the chosen magnet settings. In other words, the realistic theory presented above for Mermin's EPR experiment is local. My usage of the word 'local' is strictly in keeping with Bell's [18] and Mermin's [16] definition, i.e., local means no action-at-a-distance.

Furthermore, from the realistic viewpoint, for each possible pair of magnet settings, the probability that a particular basis state in the expression (see (1)) for the EPR state $|\psi\rangle$ is the corresponding pre-existing system spin state is given by the amplitude square of the coefficient for that basis state. Hence the probabilities of joint measurement outcomes are also given by the amplitude-squared coefficients, in exact agreement with the quantum probabilities. Therefore the local realistic theory presented above for Mermin's EPR experiment reproduces all the quantum probabilistic predictions. Locality and realism can thus coexist, contrary to conventional belief [5].

References

1. Weihs, G., et al.: Phys. Rev. Lett. **81**, 5039 (1998)
2. Rowe, M.A., et al.: Nature **409**, 791 (2001)
3. Bell, J.S.: In: Speakable and Unspeakable in Quantum Mechanics, pp. 14–21. Cambridge Univ. Press, Cambridge (1993)
4. Bell, J.S.: In: Speakable and Unspeakable in Quantum Mechanics, pp. 29–39. Cambridge Univ. Press, Cambridge (1993)
5. Aspect, A.: Nature **446**, 866 (2007)
6. Aharanov, Y., Botero, A., Scully, M.: Z. Naturforsch. A **56**, 5 (2001)
7. Hess, K., Philipp, W.: Proc. Natl. Acad. Sci. **98**, 14224 (2001)
8. Hess, K., Philipp, W.: Proc. Natl. Acad. Sci. **98**, 14228 (2001)
9. Hess, K., Philipp, W.: Europhys. Lett. **57**, 775 (2002)
10. Kracklauer, A.F.: J. Opt. B **6**, S544 (2004)
11. Lan, B.L.: J. Mod. Opt. **51**, 1069 (2004)
12. Lan, B.L.: Int. J. Theor. Phys. doi:10.1007/s10773-008-9706-1
13. Lan, B.L.: J. Russ. Laser Res. **26**, 530 (2005)
14. Lan, B.L.: J. Russ. Laser Res. **28**, 489 (2007)
15. Mermin, N.D.: Am. J. Phys. **49**, 940 (1981)
16. Mermin, N.D.: Phys. Today **38**, 38 (1985)
17. Cushing, J.T.: In: Quantum Mechanics: Historical Contingency and the Copenhagen Hegemony, p. 223. Univ. Chicago Press, Chicago (1994)
18. Bell, J.S.: In: Speakable and Unspeakable in Quantum Mechanics, pp. 139–158. Cambridge Univ. Press, Cambridge (1993)